

## Viscosity bound, causality violation and instability with stringy correction and charge

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ABSTRACT: Recently, it has been shown that if we consider the higher derivative correction, the viscosity bound conjectured to be  $\eta/s = 1/4\pi$  is violated and so is the causality. In this paper, we consider medium effect and the higher derivative correction simultaneously by adding charge and Gauss-Bonnet terms. We find that the viscosity bound violation is not changed by the charge. However, we find that two effects together create another instability for large momentum regime. We argue the presence of tachyonic modes and show it numerically. We draw a phase diagram relevant to the instability in charge-coupling space.

KEYWORDS: Black Holes in String Theory, Gauge-gravity correspondence, AdS-CFT Correspondence.

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## 1. Introduction

After the discovery of consistency of AdS/CFT [1–3] result and that of RHIC experiment on the viscosity/entropy-density ratio [4–6], much attention has been drawn to the calculational scheme provided by string theory. Some attempt has been made to map the entire process of RHIC experiment in terms of the gravity dual [7]. The way to include chemical potential in the theory was figured out in [8, 9]. Phases of these theories were also discussed in D3/D7 setup [10–12] as well as in D4D8 $\overline{D8}$  [9].

More recently, it had been conjectured that the viscosity value of theories with gravity dual may give a lower bound for the  $\eta/s = 1/4\pi$  for all possible liquid [13]. However, the authors of [14] and [15] showed that if we consider the stringy correction to  $\alpha'$  order, the viscosity bound is violated and causality is also [16] violated as a consequence (See also for more recent paper [17]).

The  $\alpha'$  terms are also related to the (in)stability issues of black holes. The instability of  $D$ -dimensional asymptotically flat Einstein-Gauss-Bonnet black holes has been discussed by several authors [18, 19]. Their results show that for the gravitational perturbations of Schwarzschild black holes in  $D =$  (from 5 to 11) Gauss-Bonnet gravity, the instability occurs only for  $D = 5$  and  $D = 6$  cases at large value of  $\alpha'$  [19].

In this paper, we add charge together with the Gauss-Bonnet term, and calculate  $\eta/s$  and consider the stability issue including the causality violation. We find that the viscosity bound violation is not changed by the charge. However, we find that for large momenta regime, there exists a new instability due to the charge effect. The linearized perturbation has a negative frequency squared signaling an instability. We draw the phase diagram relevant to the instability. We emphasize that the new instability present only if both charge and Gauss-Bonnet term present.

The rest of the paper goes as follows. In section 2, to set up we give a briefly review on the thermodynamic properties of Reissner-Nordström-AdS black brane solution in Gauss-Bonnet gravity. In section 3, the Gauss-Bonnet correction to  $\eta/s$  is calculated via Kubo formula and its charge dependence is given in an explicit form. In section 4, we study the causality violation problem for charged black branes and reproduce the results found in ref. [16]. In section 5, we discuss the stability of Reissner-Nordström-AdS black branes in Gauss-Bonnet gravity. Conclusions and discussions are presented in the last section.

## 2. Reissner-Nordström-AdS black brane in Gauss-Bonnet gravity

The thermodynamics and geometric properties of black objects in Gauss-Bonnet gravity were studied in several papers [20–25]. In this section, we mainly review the basic features of charged black holes in Gauss-Bonnet gravity. Further details can be found in [23].

We start by introducing the following action in  $D$  dimensions which includes Gauss-Bonnet terms and U(1) gauge field:

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left( R - 2\Lambda + \alpha' (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2) - 4\pi G_D F_{\mu\nu} F^{\mu\nu} \right), \quad (2.1)$$

where  $\alpha'$  is a (positive) Gauss-Bonnet coupling constant with dimension  $(\text{length})^2$  and the field strength is defied as  $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ . The corresponding Einstein equation leads

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = 8\pi G_D \left( F_{\mu\rho} F_{\nu\sigma} g^{\rho\sigma} - \frac{1}{4}g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) + T_{\mu\nu}^{\text{eff}}, \quad (2.2)$$

where

$$T_{\mu\nu}^{\text{eff}} = \alpha' \left[ \frac{1}{2}g_{\mu\nu} \left( R_{\alpha\beta\rho\sigma} R^{\alpha\beta\rho\sigma} - 4R_{\alpha\beta} R^{\alpha\beta} + R^2 \right) - 2R R_{\mu\nu} + 4R_{\mu\rho} R_{\nu}{}^\rho + 4R_{\rho\sigma} R_{\mu\nu}{}^{\rho\sigma} - 2R_{\mu\rho\sigma\gamma} R_{\nu}{}^{\rho\sigma\gamma} \right]. \quad (2.3)$$

The charged black hole solution in  $D$  dimensions for this action is described by [23]

$$ds^2 = -H(r)N^2 dt^2 + H^{-1}(r)dr^2 + \frac{r^2}{l^2} h_{ij} dx^i dx^j, \quad (2.4a)$$

$$A_t = -\frac{Q}{4\pi(D-3)r^{D-3}}, \quad (2.4b)$$

with

$$H(r) = k_0 + \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 - \frac{4\alpha}{l^2} \left( 1 - \frac{ml^2}{r^{D-1}} + \frac{q^2 l^2}{r^{2D-4}} \right)} \right), \quad \Lambda = -\frac{(D-1)(D-2)}{2l^2},$$

where  $\alpha$  and  $\alpha'$  are connected by a relation  $\alpha = (D-4)(D-3)\alpha'$  and the parameter  $l$  corresponds to AdS radius. The constant  $N^2$  will be fixed later. Note that the constant value of  $k$  can be  $\pm 1$  or 0 and  $h_{ij} dx^i dx^j$  represents the line element of a  $(D-2)$ -dimensional

hypersurface with constant curvature  $(D-2)(D-3)k_0$  and volume  $V_{D-2}$ . The gravitational mass  $M$  and the charge  $Q$  are expressed as

$$M = \frac{(D-2)V_{D-2}m}{16\pi G_D},$$

$$Q^2 = \frac{2\pi(D-2)(D-3)}{G_D}q^2.$$

Taking the limit  $\alpha' \rightarrow 0$  with  $k_0 = 0$ , the solution may correspond to one for Reissner-Nordström-AdS (RN-AdS). The hydrodynamic analysis in this background has been done in [26].

In the following, we mainly focus on five-dimensional case with  $k_0 = 0$ . Defining  $\lambda = \alpha/l^2$ , the function  $H(r)$  becomes

$$H(r) = \frac{r^2}{2\lambda l^2} \left[ 1 - \sqrt{1 - 4\lambda \left(1 - \frac{r_+^2}{r^2}\right) \left(1 - \frac{r_-^2}{r^2}\right) \left(1 - \frac{r_0^2}{r^2}\right)} \right], \quad (2.5)$$

where  $r_+$  and  $r_-$  correspond to the outer and the inner horizons, respectively, and  $-r_0^2 = r_+^2 + r_-^2$ . The constant  $N^2$  in the metric (2.4a) can be fixed at the boundary whose geometry would reduce to flat Minkowski metric conformally, i.e.  $ds^2 \propto -c^2 dt^2 + d\vec{x}^2$ . On the boundary  $r \rightarrow \infty$ , we have

$$H(r)N^2 \rightarrow \frac{r^2}{l^2},$$

so that  $N^2$  is found to be

$$N^2 = \frac{1}{2} \left( 1 + \sqrt{1 - 4\lambda} \right). \quad (2.6)$$

Note that the boundary speed of light is specified to be unity  $c = 1$ .

We shall give thermodynamic quantities of this background. The temperature at the event horizon is defined as

$$T = \frac{1}{2\pi\sqrt{g_{rr}}} \frac{d\sqrt{g_{tt}}}{dr} = \frac{Nr_+}{2\pi l^2} \left( 2 - \frac{q^2 l^2}{r_+^6} \right). \quad (2.7)$$

The black brane approaches extremal when  $q^2 l^2 / r_+^6 \rightarrow 2$  (i.e.  $T \rightarrow 0$ ). The entropy of RN-AdS black holes with Gauss-Bonnet terms can be obtained by using  $S = -\partial F / \partial T$ , where  $F$  is the free energy. After Wick rotation i.e.  $t \rightarrow i\tau$ , the free energy can be obtained from the action  $I$  in (2.1) through  $F = -TI$ . By using an identity which is derived by taking trace over the equation (2.2),

$$\alpha' (R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) + 3R - 10\Lambda - 4\pi G_5 g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} = 0,$$

the action (2.1) reduces to be on-shell,

$$\begin{aligned} I &= \frac{1}{16\pi G_5} \int_{r_+}^{\infty} dr \int_0^{\frac{1}{T}} d\tau \int d^3x \sqrt{g} \left( -2R + 8\Lambda \right) \\ &= \frac{1}{16\pi G_5} \frac{V_3 N}{T l^3} \int_{r_+}^{\infty} dr r^3 \left[ \frac{2}{r^3} (r^3 H(r))'' - \frac{48}{l^2} \right] \\ &= \frac{1}{16\pi G_5} \frac{V_3 N}{T l^3} \left[ 2 (r^3 H(r))' \Big|_{r_+}^{\infty} - \frac{12}{l^2} r^4 \Big|_{r_+}^{\infty} \right], \end{aligned}$$

where we used the scalar curvature  $R = -(r^3 H(r))''/r^3$ .<sup>1</sup> Divergent terms arise in the action and we can regulate the result by subtracting the action of the Gauss-Bonnet-modified pure AdS space, which is obtained by setting  $r_{\pm} = 0$  and  $r_0 = 0$  in  $H(r)$  (no horizons), that is to say

$$\begin{aligned} I_{\text{AdS-GB}} &= \frac{1}{16\pi G_5} \int_0^\infty dr \int_0^{\beta'} d\tau \int d^3x \sqrt{g} (-2R + 8\Lambda) \\ &= \frac{1}{16\pi G_5} \frac{\beta' V_3 N}{l^3} \left[ 2(r^3 H(r))' \Big|_0^\infty - \frac{12}{l^2} r^4 \Big|_0^\infty \right], \end{aligned}$$

where we assign a temperature  $\beta'$  to AdS space with Gauss-Bonnet terms which is [27, 15]

$$\beta' = \beta \left( \frac{g_{tt}^{\text{BH}}}{g_{tt}^{\text{AdS}}} \right)^{1/2} \Big|_{r=\infty} = \frac{1}{T} \left( \frac{g_{tt}^{\text{BH}}}{g_{tt}^{\text{AdS}}} \right)^{1/2} \Big|_{r=\infty}.$$

Then we find

$$\Delta I = I - I_{\text{AdS-GB}} = \frac{1}{16\pi G_5} \frac{12r_+^4 V_3 N}{T l^5} - \frac{V_3 N r_+^3}{2G_5 l^3}.$$

Finally, we obtain the entropy density,<sup>2</sup>

$$s = -\frac{1}{V_3} \frac{\partial F}{\partial T} = \frac{1}{V_3} \frac{\partial(T\Delta I)}{\partial T} = \frac{1}{4G_5} \frac{r_+^3}{l^3}. \quad (2.8)$$

### 3. Viscosity to entropy density ratio

Before considering a linearized perturbative theory in this background, we shall summarize the basic procedure to calculate Green function and the shear viscosity in Minkowski spacetime [28]. We work on the five-dimensional background,

$$ds = g_{mn} dx^m dx^n + g_{uu} du^2,$$

where  $x^m = (t, x, y, z)$  and  $u$  are the four-dimensional and the radial coordinates, respectively. We refer the boundary as  $u = 0$  and the horizon as  $u = 1$ . A solution of the linearized equation of motion may be given by,

$$\phi(x, u) = \int \frac{d^4k}{(2\pi)^4} e^{ikx} f_k(u) \phi^{(0)}(k),$$

where the function  $f_k(u)$  is normalized such that  $f_k(0) = 1$  at the boundary. An on-shell action might be reduced to surface terms in four dimensions by using the equation of motion,

$$I[\phi^{(0)}] = \int \frac{d^4k}{(2\pi)^4} \phi^{(0)}(-k) \mathcal{G}(k, u) \phi^{(0)}(k) \Big|_{u=0}^{u=1}, \quad (3.1)$$

<sup>1</sup>Note that when  $k_0 \neq 0$ , the scalar curvature is  $R = -(r^3 H(r))''/r^3 + 6k_0/r^2$ .

<sup>2</sup>For more general case, if  $k_0 \neq 0$ , the Bekenstein-Hawking area law is broken and the resulted entropy is given by  $\frac{V_{D-2} r_+^{D-2}}{4G_D} \left( 1 + \frac{D-2}{D-4} \frac{2\lambda^2}{r_+^2} k_0 \right)$  (see [22]).

where the function  $\mathcal{G}(k, u)$  can be written in terms of  $f_{\pm k}(u)$  and  $\partial_u f_{\pm k}(u)$ . Accommodating Gubser-Klebanov-Polyakov/Witten relation [2, 3] to Minkowski spacetime, Son and Starinets arrived at the following formulation for the retarded Green function:

$$G(k) = 2\mathcal{G}(k, u) \Big|_{u=0}, \quad (3.2)$$

where the incoming boundary condition at the horizon is imposed. In this paper we consider the tensor type perturbation in the background. By using an obtained retarded Green function, one can estimate the shear viscosity  $\eta$  via Kubo formula,

$$\eta = - \lim_{\omega \rightarrow 0} \frac{\text{Im}(G(\omega, 0))}{\omega}. \quad (3.3)$$

Now let us proceed to calculate the shear viscosity by using Green function in our background. As we see above, it is standard to introduce new dimensionless coordinate  $u = r_+^2/r^2$ . The five-dimensional metric with  $k_0 = 0$  in (2.4a) is then deformed into

$$ds^2 = \frac{-f(u)N^2 dt^2 + d\vec{x}^2}{l^2 b^2 u} + \frac{l^2 du^2}{4u^2 f(u)}, \quad (3.4)$$

where

$$f(u) = \frac{1}{2\lambda} \left[ 1 - \sqrt{1 - 4\lambda(1-u)(1+u-au^2)} \right],$$

and we denote  $a \equiv q^2 l^2 / r_+^6$  and  $b^2 \equiv 1/r_+^2$ . In this coordinate system, the event horizon of the black brane is at  $u = 1$ , while  $u = 0$  is the boundary of the AdS space.

We now study small metric fluctuation  $h_y^x(t, z, u) \equiv \phi(t, z, u)$  around the black brane background of the form

$$ds^2 = \frac{-f(u)N^2 dt^2 + d\vec{x}^2 + 2\phi(t, z, u) dx dy}{l^2 b^2 u} + \frac{l^2 du^2}{4u^2 f(u)}. \quad (3.5)$$

By considering the spin under the  $O(2)$  rotation in  $(x, y)$ -plane, gauge perturbations would be decoupled within this tensor type perturbation. Using Fourier decomposition

$$\phi(t, z, u) = \int \frac{d^4 k}{(2\pi)^4} e^{-i\omega t + ikz} \phi(k, u),$$

we can obtain the following linearized equation of motion for  $\phi(u)$  from the equation (2.2):

$$0 = \phi''(u) + \frac{g'(u)}{g(u)} \phi'(u) + \frac{\bar{\omega}^2}{uN^2 f^2(u)} \phi(u) - \frac{\bar{k}^2 [1 - 2\lambda u^2 (2u(u^{-1} f(u))'' + 3(u^{-1} f(u))')] }{u f(u) [1 + 2\lambda u^2 (u^{-1} f(u))']} \phi(u), \quad (3.6)$$

where

$$g(u) = u^{-1} f(u) [1 + 2\lambda u^2 (u^{-1} f(u))'],$$

$$\bar{\omega} \equiv \frac{l^2 b}{2} \omega, \quad \bar{k} \equiv \frac{l^2 b}{2} k,$$

and the prime denotes the derivative with respect to  $u$ .

Let us solve the equation of motion (3.6) in hydrodynamic regime i.e. small  $\omega$  and  $k$ . We first impose a solution as

$$\phi(u) = (1-u)^\nu F(u), \quad (3.7)$$

where  $F(u)$  is a regular function at the horizon  $u = 1$ , so that the singularity at the horizon might be extracted. Substituting this form into the equation of motion, we can fix the parameter  $\nu$  as  $\nu = \pm i\omega/(4\pi T)$  where  $T$  is the temperature. We here choose

$$\nu = -i \frac{\omega}{4\pi T},$$

as the incoming wave condition. In order to get the shear viscosity via Kubo formula (3.3) by using Green function (3.2), it might be sufficient to consider series expansion of the solution in terms of frequencies up to the linear order of  $\omega (= i4\pi T\nu)$ ,

$$F(u) = F_0(u) + \nu F_1(u) + \mathcal{O}(\nu^2, k^2). \quad (3.8)$$

The equation of motion (3.6) becomes the following form up to  $\mathcal{O}(\nu)$ ,

$$[g(u)F'(u)]' - \nu \left( \frac{1}{1-u} g(u) \right)' F(u) - \frac{2\nu}{1-u} g(u) F'(u) = 0. \quad (3.9)$$

Substituting the series expansion (3.8) into the equation (3.9), one can get the equations of motion for  $F_0(u)$  and  $F_1(u)$  recursively. From  $\mathcal{O}(\nu^0)$  in the equation (3.9), the equation for  $F_0(u)$  is obtained as

$$[g(u)F_0'(u)]' = 0, \quad (3.10)$$

and can be solved as

$$F_0'(u) = \frac{C_1}{g(u)},$$

where  $C_1$  is an integration constant. Regularity of  $F_0(u)$  at the horizon implies that  $C_1$  must be zero as  $g(u)$  goes to zero at the horizon. Therefore,  $F_0(u)$  is a constant,

$$F_0(u) = C, \quad (\text{const.}). \quad (3.11)$$

The solution for  $F_1(u)$  can be obtained from equation (3.9) at  $\mathcal{O}(\nu^1)$ ,

$$[g(u)F_1'(u)]' - \left( \frac{C}{1-u} g(u) \right)' = 0. \quad (3.12)$$

Integrating the above equation we get

$$F_1'(u) = \frac{C}{1-u} + \frac{C_2}{g(u)}. \quad (3.13)$$

The integration constant  $C_2$  can be fixed by the regularity condition of  $F_1(u)$  at the horizon. At the horizon  $u = 1$ , the function  $g(u)$  behaves as

$$g(u) = ((2-a)(1-2\lambda(2-a)))(1-u) + \mathcal{O}((1-u)^2).$$

Then, the regularity condition at  $u = 1$  implies

$$C_2 = -(2 - a)(1 - 2\lambda(2 - a))C. \quad (3.14)$$

The remaining constant  $C$  is estimated in terms of boundary value of the field,

$$\lim_{u \rightarrow 0} \phi(u) = \phi^{(0)},$$

so that we could fix

$$C = \phi^{(0)} \left(1 + \mathcal{O}(\nu)\right). \quad (3.15)$$

Now we shall calculate the retarded Green function. Using the equation of motion, the action reduces to the surface terms. The relevant part is given as

$$I[\phi(u)] = -\frac{r_+^4 N}{16\pi G_5 l^5} \int \frac{d^4 k}{(2\pi)^4} \left( g(u)\phi(u)\phi'(u) + \dots \right) \Big|_{u=0}^{u=1}. \quad (3.16)$$

Near the boundary  $u = \varepsilon$ , using the obtained perturbative solution for  $\phi(u)$ , we can get

$$\begin{aligned} \phi'(\varepsilon) &= -\nu \frac{(2 - a)(1 - 2\lambda(2 - a))}{g(\varepsilon)} \phi^{(0)} + \mathcal{O}(\nu^2, k^2) \\ &= i\omega \left( \frac{l^2}{2Nr_+} \right) \frac{1 - 2\lambda(2 - a)}{g(\varepsilon)} \phi^{(0)} + \mathcal{O}(\omega^2, k^2). \end{aligned} \quad (3.17)$$

Therefore we can read off the correlation function from the relation (3.2),

$$G_{xy \ xy}(\omega, k) = -i\omega \frac{1}{16\pi G_5} \left( \frac{r_+^3}{l^3} \right) \left( 1 - 2\lambda(2 - a) \right) + \mathcal{O}(\omega^2, k^2), \quad (3.18)$$

where we subtracted contact terms. Then finally, we can obtain the shear viscosity by using Kubo formula (3.3),

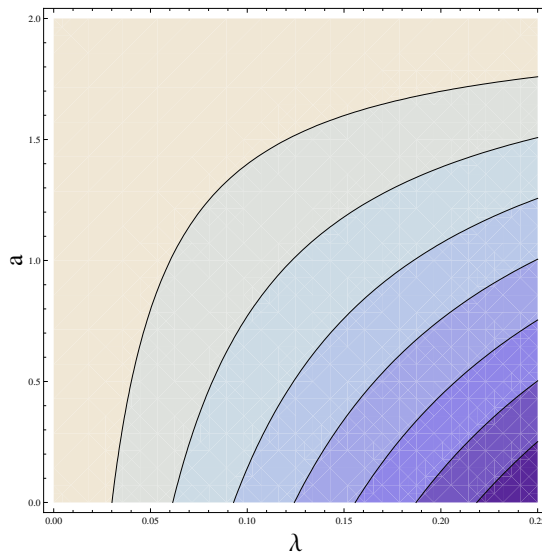
$$\eta = \frac{1}{16\pi G_5} \left( \frac{r_+^3}{l^3} \right) \left( 1 - 2\lambda(2 - a) \right). \quad (3.19)$$

The ration of the shear viscosity to the entropy density is concluded as

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 - 4\lambda \left( 1 - \frac{a}{2} \right) \right). \quad (3.20)$$

One can see explicitly that the conjectured viscosity bound can be violated for some value of  $\lambda$  and  $a$ . Figure 1 demonstrates that for fixed value of  $\eta/s$ , as the coupling constant  $\lambda$  increases,  $a$  also increases. The shear viscosity approaches zero as  $(\lambda, a) \rightarrow (1/4, 0)$  and  $\lambda$  is thus bounded by  $1/4$ . When  $a = 0$  (no charges),  $\eta/s = (1 - 4\lambda)/(4\pi)$ , we recover the result in ref. [15]. It is also worth noting that for extremal case ( $a = 2$ ), the ratio of the shear viscosity to entropy density receives no corrections from Gauss-Bonnet terms.





**Figure 1:** Shear viscosity to entropy density ratio as a function of  $a$  and  $\lambda$ . The lines correspond to  $\eta/s = 0.07, 0.06, \dots, 0.01$ , respectively, from top to bottom.

#### 4. Causality violation

It was shown that the causality could be violated if one introduced Gauss-Bonnet terms [15, 16]. We here analyze an effect of the charge to this issue.

Due to higher derivative terms in the gravity action, the equation (3.6) for the propagation of a transverse graviton differs from that of a minimally coupled massless scalar field propagating in the same background geometry. Writing the wave function as

$$\phi(x, u) = e^{-i\omega t + ikz + ik_u u}, \quad (4.1)$$

and taking large momenta limit  $k^\mu \rightarrow \infty$ , one can find that the equation of motion (3.6) reduces to

$$k^\mu k^\nu g_{\mu\nu}^{\text{eff}} \simeq 0, \quad (4.2)$$

where the effective metric is given by

$$ds_{\text{eff}}^2 = g_{\mu\nu}^{\text{eff}} dx^\mu dx^\nu = \frac{N^2 f(u)}{l^2 b^2 u} \left( -dt^2 + \frac{1}{c_g^2} dz^2 \right) + \frac{l^2}{4u^2 f(u)} du^2. \quad (4.3)$$

Note that  $c_g^2$  can be interpreted as the local speed of graviton:

$$c_g^2(u) = \frac{N^2 f(u) \left[ 1 - 2\lambda u^2 \left( 2u(u^{-1} f(u))'' + 3(u^{-1} f(u))' \right) \right]}{1 + 2\lambda u^2 (u^{-1} f(u))'}. \quad (4.4)$$

We can expand  $c_g^2$  near the boundary  $u = 0$ ,

$$c_g^2 - 1 = \left( -\frac{5}{2}(1+a) + \frac{2(1+a)}{1-4\lambda} - \frac{1+a}{2\sqrt{1-4\lambda}} \right) u^2 + \mathcal{O}(u^3). \quad (4.5)$$

As we will see below, the local speed of graviton should be smaller than 1 (the local speed of light of boundary CFT). We require

$$-\frac{5}{2} + \frac{2}{1-4\lambda} - \frac{1}{2\sqrt{1-4\lambda}} \leq 0. \quad (4.6)$$

The above equation leads to  $\lambda \leq 0.09$  without any charge dependence. This is the same result with neutral black holes in Gauss-Bonnet theory [16]. For charged black branes, the speed of graviton  $c_g^2$  is also smaller than 1 in the range  $\lambda \leq 0.09$ .

Now let us study in the regime  $\lambda > 0.09$ , to see how the causality is violated in the boundary theory. We follow the discussion in the papers [15, 16]. By using the identification  $\frac{dx^\mu}{ds} = g^{\text{eff}\mu\nu} k_\nu$ , we may rewrite the equation (4.2) into one for geodesic motion,

$$\frac{dx^\mu}{ds} \frac{dx^\nu}{ds} g_{\mu\nu}^{\text{eff}} = 0. \quad (4.7)$$

If we consider  $\omega$  and  $q$  as conserved integrals of motion along the geodesic with

$$\omega = \left(\frac{dt}{ds}\right) \frac{fN^2}{l^2 b^2 u}, \quad k = \left(\frac{dz}{ds}\right) \frac{fN^2}{l^2 b^2 u} \frac{1}{c_g^2}, \quad (4.8)$$

from (4.2) and (4.7), we can obtain

$$\left(\frac{N}{k^2 b} \frac{du^{-\frac{1}{2}}}{ds}\right)^2 = \frac{\omega^2}{k^2} - c_g^2. \quad (4.9)$$

We can simplify the above equation by recalling  $s$  as  $\tilde{s} = ks/N$  and noting that  $u = 1/(r^2 b^2)$ ,

$$\left(\frac{dr}{d\tilde{s}}\right)^2 = \alpha^2 - c_g^2, \quad \alpha^2 = \frac{\omega^2}{k^2}. \quad (4.10)$$

The geodesic equation determines the radial motion of a test particle with energy  $\alpha^2$  in an effective potential  $c_g^2$ . One can now infer that, from figure 2, geodesic line which starts at spatial infinity (the boundary) can bounce back to the boundary. In other words, the turning point appears at

$$\alpha^2 = c_g^2(u_0). \quad (4.11)$$

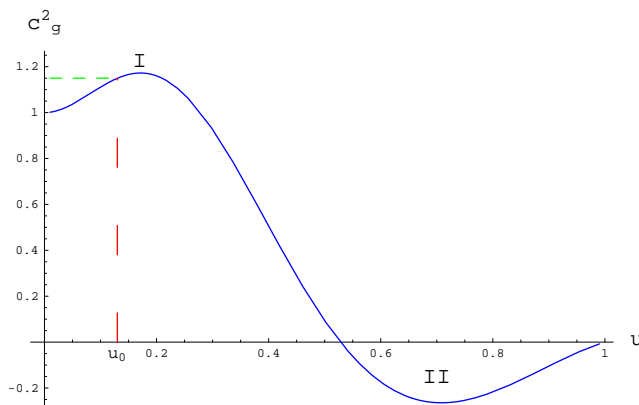
For a light-like geodesic line starting from the boundary and bounced back at the boundary, we find

$$\Delta t = 2 \int_{r_0}^{\infty} \frac{dt}{d\tilde{s}} \frac{d\tilde{s}}{dr} dr = \frac{2}{N} \int_{r_0}^{\infty} dr \frac{\alpha}{f \sqrt{\alpha^2 - c_g^2}}, \quad (4.12a)$$

$$\Delta z = 2 \int_{r_0}^{\infty} \frac{dz}{d\tilde{s}} \frac{d\tilde{s}}{dr} dr = \frac{2}{N} \int_{r_0}^{\infty} dr \frac{c_g^2}{f \sqrt{\alpha^2 - c_g^2}}. \quad (4.12b)$$

As pointed out in ref. [16], one may find microcausality violation in the boundary CFT when the a bouncing graviton geodesic satisfies  $\frac{\Delta z}{\Delta t} > 1$ . From (4.12a) and (4.12b), we can see that if we tune  $c_g(u_0)$  to be  $c_{g,\text{max}}$ , we then have

$$\frac{\Delta z}{\Delta t} \rightarrow c_{g,\text{max}} > 1. \quad (4.13)$$



**Figure 2:** The hump (I) signifies the causality violation, while the well (II) indicates that the black brane is unstable.

Since near the boundary  $c_g$  can be greater than 1, the propagation of signals in the boundary theory with speed  $\frac{\Delta z}{\Delta t}$  might become superluminal. Actually, metastable states can live in the well between  $c_{g,\max}^2$  and the boundary and  $\frac{\Delta z}{\Delta t}$  corresponds to the group velocity of metastable quasiparticles. According to the standard procedure to analyze quasinormal modes, we rewrite the wave function in a Schrödinger form,

$$-\frac{d^2\psi}{dr_*^2} + V(r(r_*))\psi = \bar{\omega}^2\psi, \quad \frac{dr_*}{dr} = \frac{1}{Nl^2b^2f(r)}, \quad (4.14)$$

where  $\psi(r(r_*))$  and the potential is defined by

$$\psi = K(r)\phi, \quad K(r) \equiv \sqrt{\frac{g(u)}{u^{-1}f(u)}} = 1 - \lambda br \frac{\partial(l^2b^2f(r))}{\partial r},$$

$$V = k^2c_g^2 + V_1(r), \quad V_1(r) \equiv N^2l^2b^2 \left[ \left( f(r) \frac{\partial \ln K(r)}{\partial r} \right)^2 + f(r) \frac{\partial}{\partial r} \left( f(r) \frac{\partial \ln K(r)}{\partial r} \right) \right].$$

Following the procedure of ref. [16], one can find that the group velocity of the graviton is given by

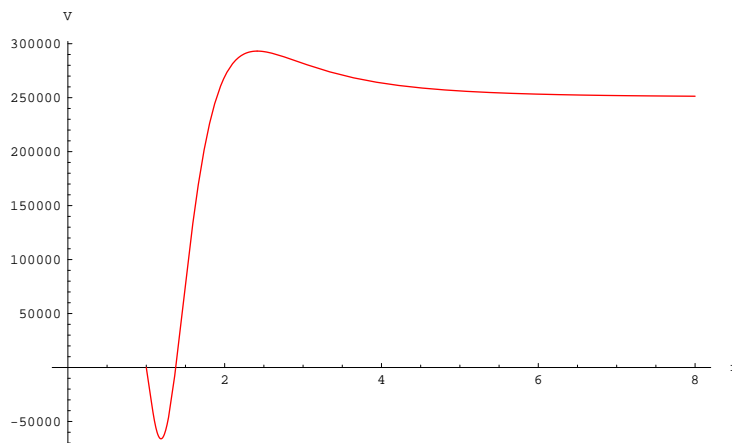
$$v_g = \frac{d\omega}{dk} = \frac{\Delta z}{\Delta t}. \quad (4.15)$$

Therefore, signals in the boundary theory propagate outside of the light cone. Again, we confirm that microcausality violation happens in the CFT. One may expect that when  $\lambda \leq 0.09$ , the theory with Gauss-Bonnet corrections is safe and consistent.

## 5. Instability

Apart from the causality violation, for RN-AdS black brane in Gauss-Bonnet theory, the charges give the instability of the black brane within the window of  $0 < \lambda \leq 0.09$ .

From figure 3, we can see that the Schrödinger potential develops a negative gap near the horizon. We will now show that in the large momentum limit, the negative-valued



**Figure 3:** Schrödinger potential  $V(r)$  as a function of  $r$  for  $\lambda = 0.2$ ,  $a = 1.7$  and  $k = 500$ . Near the horizon, the potential develops a negative-valued well and then negative-energy bound states will appear. Near the boundary, the potential develops a hump which corresponds to superluminal propagation of metastable quasiparticles.

potential leads to instability of the black brane. In the large momenta limit  $k^\mu \rightarrow \infty$ , the dominant contribution to the potential is given by  $k^2 c_g^2$ . For near extremal cases,  $c_g^2$  can be negative near the horizon and  $V \simeq k^2 c_g^2$  can be deep enough (see figure 3). Thus bound states can live in the negative-valued well. The negative energy bound state corresponds to modes of tachyonic mass on Minkowski slices [29] and signals an instability of the black brane [18, 19]. Let us expand  $c_g^2$  in series of  $(1 - u)$ ,

$$c_g^2 = \frac{(2 - a)(1 + 4\lambda - 14a\lambda - 32\lambda^2 + 32a\lambda^2 - 8a^2\lambda^2)}{(1 - 4\lambda + a\lambda)}(1 - u) + \mathcal{O}((1 - u)^2). \quad (5.1)$$

Since  $0 \leq a \leq 2$ , and  $0 \leq u \leq 1$ ,  $c_g^2$  will be negative, if

$$\frac{(1 + 4\lambda - 14a\lambda - 32\lambda^2 + 32a\lambda^2 - 8a^2\lambda^2)}{(1 - 4\lambda + 2a\lambda)} < 0. \quad (5.2)$$

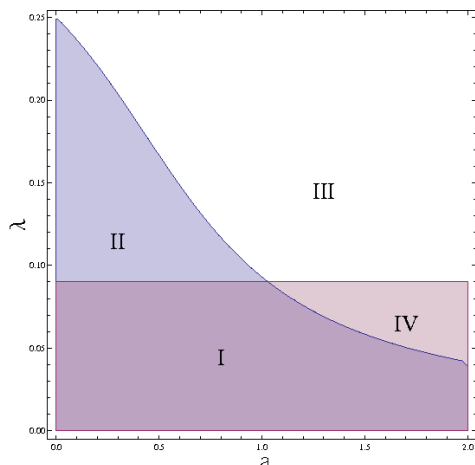
From the above formula, we find the critical value of  $\lambda$ ,

$$\lambda_c = \frac{2 - 7a + \sqrt{3}\sqrt{12 - 20a + 19a^2}}{8(a - 2)^2}. \quad (5.3)$$

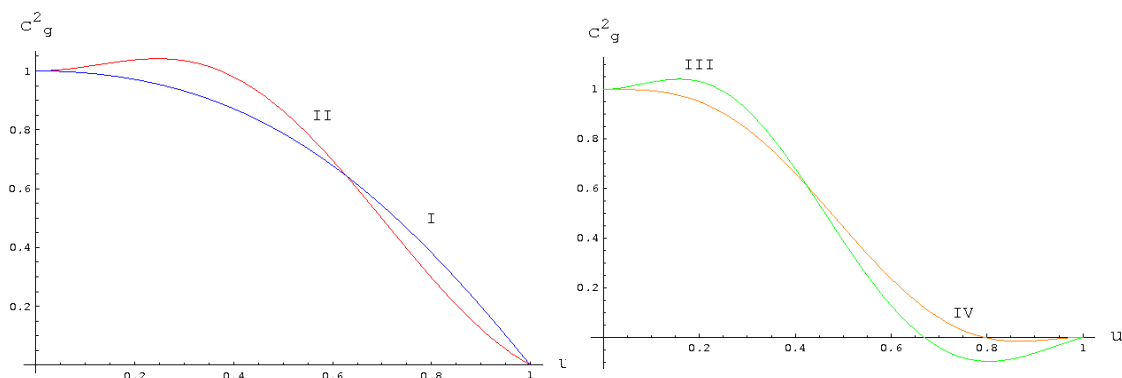
Above the line of  $\lambda_c$ ,  $c_g^2$  can be negative (see figure 2). The minimal value of  $\lambda_c$  can be obtained in the limit  $a \rightarrow 2$ ,

$$\lambda_{c, \min} = \frac{1}{24}. \quad (5.4)$$

Figure 4 shows us that the two lines  $\lambda_c(a)$  and  $\lambda = 0.09$  separates the physics into four regions in  $(a, \lambda)$  space. The physics in region I so far can be consistent. In region II, causality violation can be found. In region III, causality violation as well as unstable quasinormal modes (QNMs) appear. In region IV, we can only find unstable QNMs.



**Figure 4:** Phase diagram for the instability in  $a$ - $\lambda$  space. Region I: There are no causality violation and bulk bound states. Region II: Causality violation can happen, but bound states do not appear in the bulk. Region III: Both causality violation and instability happen in this region. Region IV: There is no causality violation, but the black brane is unstable.



**Figure 5:**  $c_g^2$  as a function of  $u$ . Line I describes the behavior of  $c_g^2$  in region I for  $\lambda = 0.05$ ,  $a = 0.2$ . Line II shows that  $c_g^2 > 1$  at some value of  $u$  near the boundary for  $\lambda = 0.14$ ,  $a = 0.4$ .

**Figure 6:**  $c_g^2$  as a function of  $u$ . Line III:  $c_g^2$  has a hump greater than 1 and a negative-valued well (We set  $\lambda = 0.15$ ,  $a = 1.7$ ). Line IV:  $c_g^2$  only has a negative-valued well ( $\lambda = 0.089$ ,  $a = 1.99$ )

Figures 5 and 6 show us explicitly the behaviors of  $c_g^2$  in different regions. In order to demonstrate that the peculiar feature of  $c_g^2 < 0$  signals the instability of the RN-AdS black brane in Gauss-Bonnet theory, we solve the Schrödinger equation (4.14) with negative-valued potential numerically and find some unstable QNMs (see table 1). From table 1, we can find that the real part of  $\omega$  is vanishing, while the imaginary part of  $\omega$  is positive. Inserting the values of  $\omega$  into the equation (4.1), one can see that gravitational instability grows as time goes on and then the black brane becomes unstable against gravitational perturbation. The numerical analysis also indicate that the black brane becomes stable under gravitational perturbation, when we restrict  $\lambda$  to be  $\lambda \leq 1/24$ .

$\lambda$	$a = 1.9$	$a = 1.7$	$a = 1.4$	$a = 1.1$	$a = 0.8$	$a = 0.5$
0.2	258.2823 <i>i</i>	255.9158 <i>i</i>	245.6454 <i>i</i>	223.4933 <i>i</i>	179.9353 <i>i</i>	94.4075 <i>i</i>
0.15	158.0242 <i>i</i>	154.8652 <i>i</i>	141.6929 <i>i</i>	113.4577 <i>i</i>	60.5891 <i>i</i>	—
0.1	79.3897 <i>i</i>	75.4296 <i>i</i>	59.4755 <i>i</i>	26.9101 <i>i</i>	—	—
0.07	33.9841 <i>i</i>	29.3078 <i>i</i>	—	—	—	—
0.05	6.6761 <i>i</i>	—	—	—	—	—

**Table 1:** Unstable QNMs for charged GB black brane perturbation of tensor type.

## 6. Conclusions and discussions

In summary, we have computed the charge dependence of  $\eta/s$  for Gauss-Bonnet theory. We have taken RN-AdS black brane solution into Gauss-Bonnet gravity and used the Kubo formula to compute the viscosity of the dual boundary theory. The ratio of the shear viscosity to entropy density was found to be  $\eta/s = \left(1 - 4\lambda(1 - a/2)\right)/(4\pi)$ , which violated the conjectured viscosity bound for non-extremal RN-AdS black brane in Gauss-Bonnet gravity. However, for extremal case, the conjectured lower viscosity-entropy density bound  $1/4\pi$  can be recovered.

The causality violation and the instability of charged black brane were also analyzed in this paper. We have confirmed the results found in previous work that when  $\lambda > 0.09$ , causality violation happens in the boundary CFT [16]. It is interesting to notice that charges introduce instability of the black brane even in the range  $0 < \lambda \leq 0.09$ .

Our final remark is about the interpretation of the charge effect. If one introduces the bulk filling D-branes, one can consider the two kind of Maxwell fields, one for  $R$ -charge  $U(1)$  and the other for the baryon charge for the brane gauge field [30]. Since both fields couple to the gravity in the same way, one can consider the charge in our analysis either as the  $R$ -charge or baryon charge. For the latter case, we can interpret the charge effect as the effect of finite baryon density.

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